Mirror, Mirror, On the Wall Part 2

Testing of The ATM Round Robin Mirrors

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I. Overview of Tests Performed

Listed here are the tests that I performed on these mirrors. The pertinent experimental details and the results of each test are discussed below.

The first two tests are intended to assess the quality of the glass used to make the mirror and the quality of the work leading up to the figuring of the mirror, respectively. They are:

1) The Crossed Polarizer Test for Strain
2) The Laser Test for Completeness of Polish

The remaining tests are intended to assess the figure of the mirror. They are:

3) The Ronchi Test
4) The Zonal Foucault Test
5) The Halo-Balance Measurement to Extend the Foucault Test
6) The Couder (Knife Edge Diffraction) Test for Edge Condition
7) The Eyepiece test for Astigmatism

II. The Crossed Polarizer Test for Strain

A. Experimental Procedure

A source of highly polarized light with an area larger than that of the mirrors being tested was constructed as follows. First, two 8 x 11” sheets of polaroid filter material were obtained from Surplus Shed. The polarization axes of the two sheets were found to be identical. These two filters were taped side by side onto the white plastic diffuser of a light box normally used for viewing transparencies. Light not passing through the polarizing filters was masked off.

A photographic quality polarizing filter was used as the analyzer. As usual, it was held in front of the eye or camera and rotated to achieve the maximum darkening of the light coming directly from the source.

Each mirror under test was oiled on the back to minimize de-polarization of the light passing through the unpolished back of the mirror and then placed between the light source and the analyzer. For all 3 test mirrors, diagonal 1 was oriented horizontally and the back of the mirror was placed nearest the light source.

With the room lights darkened, the mirrors were visually examined and photographed for evidence of strain indicative of unwanted stresses within the glass.

An Olympus C3040 digital camera mounted on a tripod was used to photograph the strain patterns. The exposure was manually controlled and set to achieve a brightness level that reproduced what was seen under visual examination. A piece of manila file folder placed at the side of the mirrors was used as a reference for determining the exposure.

The oil was removed from the backs of the mirrors by repetitive wiping with paper towels saturated with isopropyl alcohol.

B. Comments on the Crossed Polarizer Test

A section discussing strain in glass in ATM 1, revised, page 498, ends: "Dark and light streaks indicate bad strains: reject glass. Faint shadings probably are inconsequential." Texereau, 2nd edition page 147, implies that a mild Maltese cross pattern represents a tolerable level of strain in his discussion on coring Cassegrain primaries. However, that discussion hinges on whether the glass might shatter when cored and does not explicitly address the issue of the figure changing due to strain.

I believe that strain patterns that are spread over or divide large areas of the mirror are of primary concern. Differential expansion and contraction of strained glass joining two large areas of a mirror could act as a sort of hinge around which the mirror bends in response to temperature changes. Strain affecting large areas of the glass is usually detectable under crossed polarizers as light and dark bands. Strain produced by small inclusions in the glass is also visible, but is less likely to damage the overall figure of the mirror, though it might produce localized hills or holes in the mirror's figure. Highly localized strain such as that associated with very small inclusions is thus of little concern because the area potentially affected is very small.

C. Results of the Crossed Polarizer Test

Mirror A showed no indications of strain affecting large areas of the mirror. Several inclusions and marks on the back of the mirror showed visible but highly localized strain. An inclusion of what appeared to be a strand of a foreign material approaching 3 inches in length showed highly visible evidence of strain, but I suspect that this occurs at too localized a depth within the glass to noticeably affect the mirror's figure. Consequently, I would not expect Mirror A to show any temperature induced changes of figure due to strain.

Mirror B showed a slight Maltese cross pattern. Also, there is one inclusion of obviously foreign material about 1/8" square around which pronounced strain is evident. There is also evidence of strain associated with what appear to be long strands embedded within the bulk of the glass. Perhaps the pour was interrupted while foreign matter was removed and thin strands of glass were left behind that were embedded when the pour was completed. In any case, the stresses associated with all of these inclusions are probably localized enough not to threaten the figure of Mirror B in actual use.

Mirror C showed a slightly more pronounced Maltese cross pattern than Mirror B, indicating a higher level of overall strain. It would be interesting to test the figure of this mirror at a temperature characteristic of winter observing to obtain some experimental evidence as to whether the strain in this mirror is acceptable or not.

Unfortunately, there is very little by way of scientific evidence indicating precisely the severity and type of strain which might be acceptable in a telescope mirror. Perhaps these mirrors will provide a few much needed data points in this regard.

III. The Laser Test for Completeness of Polish

A. Experimental Procedure

This test was conducted using a <5mW class IIIA "high brightness" red LED laser pointer obtained from Calpac lasers (calpaclaser.com). The room was darkened and the mirrors and reference glass were placed on black flocking paper to maximize the visual detectability of scattering from the surface of the glass. The laser beam was directed onto the glass at an angle of 45 degrees from the normal. The light scattered at the surface of the mirror was viewed at 90 degrees relative to the plane defined by the path of the laser beam and its reflection, and at about 45 degrees relative to the face of the mirror. Scattering was observed from a distance of about 12".

A 10" diameter 1" thick piece of pristine soda-lime float glass was used as a reference to provide a visual standard of "good polish". Scattering from the reference surface was of course faint, but it was nevertheless easily detected visually.

There is a non-zero lower limit to the number of imperfections per unit area that can be eliminated by polishing that is characteristic of the glass used. When that limit is achieved, the glass can be said to have reached "best polish". I've read conflicting reports on the question of whether plate glass or Pyrex has the smallest number of defects per unit area at best polish. So, I'm not sure whether a float glass standard is too stringent or lenient of a standard for comparison. In addition, it's possible that float glass, which is melted to achieve a clean surface and not actually polished, may show fewer defects per unit area than can be achieved by polishing the same glass. In any case, I didn't have a thick enough piece of highly polished Pyrex to use as a reference so I used float glass.

As dust can rarely be completely removed from a mirror without taking extreme precautions, it is essential to have some means of discerning whether scattering at the surface is due to dust or surface imperfections. On a mirror that has been reasonably well cleaned, increased scattering caused by dust will falloff to the average rapidly as the beam is moved along a particular radius of the mirror because most of the surface will be dust free. In contrast, scattering due to microscopic pits will change very little along a particular radius of a mirror. Isolated large pits can give a bright return just like dust, but large pits are detectable by direct visual examination using a magnifier and would normally be found and eliminated during fine grinding. Further, if there is suspicion that moderately large pits are present, the mirror can be cleaned again and the mirror re-tested. Dust will appear to move around while large pits will remain stationary.

So, to differentiate between scattering caused by dust and scattering caused by pits and other defects on the surface of the glass, the laser spot was manually scanned back and forth across the diameter of each surface to be tested. When an unusually bright spot was found, the beam was moved along an arc at the same distance from the center of the mirror. Isolated, brighter returns were ignored as dust.

B. Results of the Laser Test for Completeness of Polish

The amount of scatter seen on all 3 test mirrors was observed to be highly independent of distance from the center of the mirror, indicating that the polish on all 3 mirrors was uniform. However, all 3 test mirrors showed more surface scatter than the float glass reference. Mirrors B and C showed about twice the scatter observed for the reference glass, while mirror A showed about 4 times the scatter of the reference. Though these numbers are based on a visual estimate and therefore cannot to be taken as quantitative, the differences noted are easily seen and very real.

Superficially, none of the three test mirrors showed the strong green coloration characteristic of soda-lime or float glass, so all three are assumed to be some variant of Pyrex. Assuming that all three test mirrors are made from the same type of Pyrex, we can conclude that fine pits have definitely not been completely polished out of Mirror A. It is also possible that Mirrors B and C are not completely polished out. However, the higher scattering intensity seen with Mirrors B and C as compared to the reference may be due to the limitations of either the glass used or the polishing process itself.

IV. General Conditions Used for Mirror Figure Tests

All tests for assessing the figure of the mirrors were conducted in a dimly lit basement laboratory. Air conditioning was shut off prior to testing to control air currents and testing proceeded only after the effects of air currents were verified to be visually undetectable.

The test mirrors were handled while wearing clean Playtex “Hand-Saver” rubber gloves to minimize heat transfer.

A mirror test stand was used that supported the back of the mirrors at three points near the edge and supported the weight of the mirrors using a sling.

For the light source, a 30 micron slit was illuminated by a 2600 milli-candela (mcd) blue LED with the integral plastic lens intact. The lens molded into the packaging of the LED reduces the divergence of the light beam that proceeds toward the slit and thus helps in getting more light through the slit. A small section of glass cut from a microscope slide which was ground with 5 micron AlOx on one side was used between the LED and the slit to act as a diffuser. The use of a diffuser does not completely negate the effect of the lens on the LED, because light passing through a translucent diffuser passes through with its general direction largely unaltered.

The position of the source was fixed, not moving, unless specifically stated otherwise.

Longitudinal and lateral knife edge motion was achieved using surplus micrometer driven stages in an X-Y configuration. Facing the mirror under test, the knife edge is driven into the beam from the right and the light source is offset about 1.25” to the left. The longitudinal micrometer head was read for both the Foucault zonal readings and the Halo-Balance readings, and it was set as required for the Ronchi test. By estimating the value of the last digit, the longitudinal micrometer head was read to 4 digits past the decimal point. All 4 digits were noted when reading the micrometer since in some instances rounding might introduce significant errors.

For the Foucault and Halo-Balance tests, a viewing telescope of about 4 power was attached to the stage carrying the knife edge. A Plossl eyepiece which made the images of the mirrors under test fill about 50% of the visible field of view was used.

All numerical error values are calculated as wavefront errors, even when the discussion refers to some particular part of a mirror surface.

V. The Ronchi Test

A. Experimental

The Ronchi test was used to assess the general condition of each mirror and to gauge the condition of the edges of the three mirrors. No attempt was made to quantitatively assess the degree of correction for spherical aberration applied to the mirrors using the Ronchi test, though in some cases high and low zones are described as having too little or to much correction relative to the rest of the mirror.

The views seen in the Ronchi test were photographed using an Olympus C3040 digital camera mounted on the longitudinal test stage immediately behind the film grating. The exposure was manually controlled and set to minimize loss of detail at the edge of the mirror.

The Ronchi test was conducted in both Slit-Grating mode and in Grating-Grating mode.

In Slit-Grating mode, the previously described fixed slit source was used together with a 133 line per inch film grating, which was aligned to the focussed image of the slit. The lateral source to grating distance in this arrangement was about 1.25”. Observations and photographs were made for all mirrors at the following longitudinal offsets relative to the central radius of curvature (ROC): -0.5”, -0.3”, at ROC, +0.3”, +0.5”. Negative offsets indicate that the grating was closer to the mirror than the ROC. Positive offsets indicate that the grating was farther from the mirror than the ROC.

To assist in the assessment of the edges, a ring mask obscuring the outer 0.25” of each mirror was placed over each mirror to provide a reference image complete with diffraction effects, but without a significant edge problem. The mirrors were photographed with these masks at all offsets except at the ROC.

In Grating-Grating mode, a 2600 mcd blue LED was placed directly behind the same film grating used for viewing and about 1.25” above the area of the grating through which the return beam passed. Consequently, in this mode the source was no longer fixed, but moving. A diffuser is not needed in Grating-Grating mode and was not used. In this mode, observations and photographs were made for all mirrors at the following longitudinal offsets relative to the central radius of curvature (ROC): -0.25”, -0.15”, at ROC, +0.15”, +0.25”. The offsets used were exactly half of those employed in Slit-Grating mode to compensate for the effect of the moving source. In this way, images with comparable numbers of Ronchi bands were obtained using both the slit-grating and grating-grating modes of the Ronchi test.

B. Results of the Ronchi Test

Mirror A was obviously rough, with a clear turned-down edge about 5/16” wide. The roughness alone is enough to rate this mirror as incomplete.

Mirror B was generally smooth, but showed an abrupt change in the curvature of the Ronchi bands at about 1.125” in from the edge indicative of a slightly high zone. To the extent that it can be judged using the Ronchi test, Mirror B has a perfect edge.

Mirror C was generally smooth, but showed a narrow turned-down edge about 0.1” wide. It also showed a very slightly raised central area a little under 1” in diameter.

VI. The Zonal Foucault Test Using the Couder Masks Provided

A. Experimental

The zonal Foucault test was performed on all 3 mirrors using the supplied Couder masks. The masks provided were centered on the mirrors by first taping a set of four tabs cut from aluminum flashing to the sides of each mirror. The tabs were placed at 45, 135, 225, and 315 degrees relative to the top of the mirror so as not to block the zone openings at the edge of the mask. After the mask was placed on the mirror, the tabs were bent inward to constrain and center the mask. When the dark cardboard zone isolator masks were in place, the Couder mask could not be accurately centered over the mirror owing to slight differences in the diameters of the two masks, so the isolator masks were not used.

The inside and outside zone radii used in the calculation were measured from the masks provided using a dial indicator, with the exception of the outer radius of the outermost zone. The radius of each of the three mirrors was found to be smaller than the outer radius of the outermost zone measured from the matching Couder mask, so the radius of each mirror was used instead.

Successive Foucault knife edge readings were made by alternately first brightening the left and then the right opening of a zone pair to balance any tendency to stop consistently short or long of a true null. However, the null was not always approached in a strictly monotonic fashion since in many cases the adjustment of the knife edge would go through the null and brighten the opposite opening slightly before the imbalance was visually detected.

The knife was driven laterally in and out of the beam during each null reading to find the point of maximum sensitivity and as a cross-check to verify that an acceptable null position had indeed been reached in each case. This is particularly important for the innermost zones. On occasion, withdrawing the knife edge very slightly has made it evident that the knife edge had not yet reached a true null position. The best visual sensitivity was found to occur at the point where the central areas of each zonal openings were very dim, but bright enough to be seen as blue in color.

The raw knife edge readings and other data required to perform zonal Foucault analysis appear in Appendix I.

The Foucault knife edge readings were processed with 2 different freeware programs written for this purpose: Tex version 3.2b, by Michael Lindner, Larry Phillips, and Dale Eason; and Figure 4.5, written by Dave Rowe. I also ran the data through Sixtests for Windows, written by Jim Burrows. However, I feel that my use of Sixtests is more error prone due to the need to convert the raw data from inches to mm prior to entry, so I have not relied on Sixtests in making my assessment of the test mirrors.

B. Results

Following Texereau's manual calculation procedure, Tex 3.2b accepts knife edge data on two diameters, then averages that data and uses it to calculate the P-V wavefront error. Figure 4.5 accepts data on only 1 diameter at a time, but calculates several area weighted criteria including the Strehl ratio and the RMS error in addition to the usual P-V error on the wavefront.

Mirror A

Tex 3.2b Results:

P-V error on the wavefront = 1/3.6 waves.
Zone 1 deviates the most from the best-fit parabola.

Figure 4.5 Results:

Diagonal 1: P-V wavefront error = 1/3.0; Strehl ratio = 0.875; RMS = 1/16.8
Diagonal 2: P-V wavefront error = 1/4.3; Strehl ratio = 0.924; RMS = 1/22.1
Zone 1 deviates the most from the best-fit parabola.

One interesting point about the Figure 4.5 results for diagonal 1 is that the P-V rating for this mirror doesn't quite reach the 1/4 wave Rayleigh criterion, yet the Strehl ratio is noticeably better than the 0.8 value considered a match for 1/4 wave P-V error. This is possible when a central zone representing a small area of the mirror shows the greatest deviation from the best-fit parabola, as is the case here.

Mirror B

Tex 3.2b Results:

P-V error on the wavefront = 1/8.7 waves.
Zone 4 deviates the most from the best-fit parabola.

Figure 4.5 Results:

Diagonal 1: P-V wavefront error = 1/11.4; Strehl ratio = 0.971; RMS = 1/36.9
Diagonal 2: P-V wavefront error = 1/6.12; Strehl ratio = 0.925; RMS = 1/22.6

Zone 4 deviates the most from the best-fit parabola.

The rather large difference between the Strehl ratio for the two diameters of Mirror B hints at the possibility of astigmatism in this mirror, though that is not the only potential explanation for this behavior.

Mirror C

Tex 3.2b Results:

P-V error on the wavefront = 1/14.8 waves.
Zone 1 deviates the most from the best-fit parabola.

Figure 4.5 Results:

Diagonal 1: P-V wavefront error = 1/21.7; Strehl ratio = 0.994; RMS = 1/81.7
Diagonal 2: P-V wavefront error = 1/10.3; Strehl ratio = 0.989; RMS = 1/58.8
Zone 1 deviates most from the best-fit parabola.

One interesting point about the Figure 4.5 results for the two diameters of Mirror C is the rather large difference in P-V error as compared to the area weighted RMS error and Strehl ratio.

VII. The Halo Balance Extension to the Foucault Test

A. Abstract

This section details a new technique for obtaining a zonal measurement at the extreme edge of a mirror based on measuring the longitudinal knife edge position at which the left and right sides of the Rayleigh diffraction ring have equal intensities. Including a zonal measurement at the extreme edge of the mirror improves the accuracy of the Foucault analysis by placing a zonal measurement on the most often troubled part of the mirror and by preventing the data reduction procedure from assuming that the otherwise unmeasured outer edge of the mirror does whatever is needed to minimize the effect of errors found on the measured interior of the mirror.

B. The Rayleigh Diffraction Ring or Halo

When the Foucault test is conducted without a mask and the knife edge is moved laterally into the beam near the radius of curvature (ROC) of the mirror, to the point that the entire mirror surface appears dark, there remains at the extreme edge of the mirror a brightly illuminated ring or halo which can only be extinguished by driving the knife edge considerably farther into the beam. This is the Rayleigh diffraction ring, also known as a halo in the professional optics literature. On a mirror with a severely turned edge (up or down) it may be impossible to see a complete ring at any longitudinal knife edge setting.

C. Prior Tests Based on the Rayleigh Diffraction Ring

Checking for a balance in the left and right sides of the Rayleigh diffraction ring has been suggested in messages on the ATM mailing list from time to time as a test for a turned down edge. However, in searching the ATM list archives for more information on this test, I found no indication of the longitudinal knife edge position at which the two sides of the diffraction ring should balance if the edge of the mirror is good, nor did I find any indication of the magnitude of the wavefront error that can be considered detectable with this test.

A test for detecting "Slight Turned Edge" using half of the Rayleigh diffraction ring was described by A.W. Everest in the revised edition of Amateur Telescope Making 1, A.G. Ingalls ed., 464-465, Willmann-Bell (1996). Instead of comparing the brightness from both sides of the diffraction ring, Everest compared the side of the diffraction ring opposite the knife edge with the diffraction from a straightedge placed in front of the mirror. Equal dimming of the diffraction effects as the knife edge was moved laterally into the beam was said to indicate a good edge. If diffraction from the straightedge dimmed first, a turned down edge was indicated.

Unfortunately, there are some inconsistencies and missing information in the ATM1 discussion of the Everest test. In particular, Everest gave no indication as to the particular longitudinal knife edge position at which the test should be conducted, nor any indication of the magnitude of the wavefront error that can be detected. He also failed to mention that the entire diffraction ring and not merely half of it can be seen. Further, Everest stated that the semi-circular illumination of the edge persists over several inches of knife edge travel, and "only most careful workmanship by an experienced workman will eliminate it entirely." Everest is clearly in error here, as the Rayleigh diffraction ring can never be eliminated by working the mirror. It is a fundamental property of light itself and will appear forever on good and bad mirrors alike. One can move the knife edge longitudinally so far from the correct position that the diffraction ring is no longer visible, but this has nothing to do with the workmanship on the mirror.

As will be made evident shortly, neither of these tests can be relied on without first correctly setting the longitudinal knife edge position, yet all accounts of these tests have failed to specify the longitudinal knife edge position required for the test to work.

To date, there has been little explanation in the ATM literature of how either of these tests work, nor any satisfactory explanation of the Rayleigh diffraction ring itself.

D. An Explanation for the Rayleigh Diffraction Ring, or Halo

After searching through several physics textbooks and a number of books on the subject, including R.W. Ditchburn's "Light", 4th Edition, Dover (1991), I was surprised at not finding an explanation of the Rayleigh diffraction ring. However, study of the diffraction ring itself has led me to a basic understanding which I will attempt to relate here.

In a nutshell, here's how the Rayleigh diffraction ring works. The light reflected near the extreme edge of the mirror is spread by diffraction over a slightly wider range of angles than normally dictated by the width of the source slit and the geometry of the Foucault test. Importantly, diffraction spreads the light equally to each side of the reflected rays coming from both the left and right edges of the mirror, so light from both sides of the mirror can get past the knife edge. The diffracted light that gets past the knife edge when all of the undiffracted light returning from the mirror is blocked is what we see as the Rayleigh diffraction ring.

A key characteristic of this diffraction ring, or halo, is that the paths of the light for both the left and right sides of the halo are principally determined by the geometry of the Foucault test. Diffraction changes this only to the extent that it spreads the light out a little and effectively shifts the visible portion to one side of the knife edge. Consequently, we can treat the left and right sides of the diffraction halo as the two sides of just another zone in the Foucault test, albeit a very narrow one located at the extreme edge of the mirror. To isolate this extreme edge zone from the rest of the mirror we need only to drive the knife edge laterally a little further into the returning beam than we would normally set it for the Foucault test. No mask or pinstick is needed.

In addition, it should be possible to make measurements of any very narrow zone on the mirror by simply superimposing a ring mask of the desired radius and determining the knife edge position at which the resulting diffraction halo comes into balance.

The zonal nature of the diffraction halo is the main reason why we sometimes only see an imbalanced or incomplete diffraction ring. We're simply at the wrong longitudinal knife edge position to see both sides of the zone in balance.

Importantly, it should be noted that there are conditions under which it is impossible to balance the left and right sides of the diffraction ring by shifting the longitudinal position of the knife edge. In particular, this occurs when the extreme edge of the mirror has a very steeply changing ROC characteristic of a sharply turned edge.

E. Using Halo-Balancing with Existing Data Reduction Procedures

Ideally, all existing Foucault data analysis procedures would correctly utilize the edge measurements obtained by halo-balancing without modification. In fact, only some do. In particular, any version of Sixtests or Figure will work, while Tex and the manual procedure specified in Texereau's book will not.

Tex doesn't work because it calculates the weight given to each zone from the inner and outer radii. Since the inner and outer radii associated with the halo-balancing measurement of the edge zone are virtually equal, Tex will effectively give the edge zone a weight of zero. While artificially changing all of the zonal radii in the test could address the weighting problem, doing so would lead Tex to incorrectly calculate the zone centers. Consequently, there is no way to coax existing versions of Tex to utilize the edge data correctly.

The manual data reduction procedure specified in Texereau's book also will not properly utilize the edge data obtained by halo-balancing. The graphical integration procedure used by Texereau automatically weights the results of each zonal measurement in proportion to the width of each zone. Since the inner and outer radii of the edge zone are virtually identical, the edge readings obtained using halo balancing are given no weight. Since this is a manual procedure, an alternative procedure or a fix can obviously be devised.

F. Experimental Test of Halo Balancing for Zonal Measurements

To test the idea that a diffraction halo could be treated as a zone in the Foucault sense, I did an experiment using Mirror B in which I measured the halo balance offsets using a set of ring masks that were cut to match the area-weighted zonal radii of the Couder mask provided for the Round Robin testing. The radii of the area-weighted zone centers were calculated according to the equation $r = [(r_i^2 + r_o^2)/2]^{1/2}$. If halo balancing yields results that are equivalent to the zonal Foucault test, as predicted above, the means of the halo balance readings obtained using the ring masks should not be significantly different from the means obtained using the Foucault readings taken from matching zones of the mirror.

The means (averages) of the Foucault readings and of the halo balance readings are compared in the table below. Data from the “extreme edge” zone, which can only be measured using halo balancing, is also included and was designated as zone 7.

Mirror B Diagonal 1 Knife Edge Readings, in Inches

Zone	Foucault Readings (n=6)		Halo Balance Readings (n=12)		Difference (Halo - Foucault)
	Mean	Std. Dev.	Mean	Std. Dev.	
1	-0.8547	0.0016	-0.909	0.032	-0.054
2	-0.8200	0.0028	-0.845	0.011	-0.025
3	-0.7975	0.0018	-0.813	0.013	-0.017
4	-0.7642	0.0010	-0.774	0.0066	-0.010
5	-0.7155	0.0030	-0.721	0.0084	-0.0055
6	-0.6944	0.0011	-0.701	0.019	-0.0066
7			-0.650	0.022	

While the two sets of means are generally in good agreement, noteworthy differences can be seen for zones 1 through 4 near the center of the mirror. The fact that the signs of these differences are uniformly negative suggests that halo balancing may have a slight tendency to underestimate the ROC. However, the differences diminish markedly from center to edge, falling to 0.010” at zone 4. At zone 6, the outermost zone that can be measured using the Foucault test, the two means agree to better than 0.007”.

Before addressing the issue of whether a difference of 0.007” is adequate agreement, it is worthwhile to first ask whether this difference is significant in a statistical sense. There is a relatively involved statistical significance test called the Welch-test which accounts for the fact that the variances of the two sets of data to be compared are not equal. However, there is no need here for an exact statistical test. If we find that the mean for the Foucault measurements falls within the confidence bounds that we calculate for the mean of the halo balancing measurements, the difference between the two means is certain to be found insignificant using a more exact test. The disadvantage of this simplification is that we might find a difference to be significant when it is not. That’s acceptable in this application.

The confidence bounds for the mean obtained from n measurements can be calculated from the equations:

$$\text{upper limit} = \text{mean} + t\, s / \text{sqrt}(n)$$
$$\text{lower limit} = \text{mean} - t\, s / \text{sqrt}(n).$$

Here, t is the value of Student's t-statistic for an alpha of .025 and n-1 degrees of freedom (t=2.201), s is the standard deviation of the readings (.019"), and n is the number of readings from which the mean and standard deviation were calculated (n=12). Values of the t-statistic can be found from tables in virtually all statistics textbooks.

The calculated bounds (+,- 0.012") bracket the range of values over which we are 95% certain that the population mean for the halo balance measurement falls for zone 6. In practice, that says that if we could make an infinitely large number of measurements from which to calculate the mean, there is a 95% chance that the resulting mean would fall within this range. The mean for the Foucault measurements at zone 6 differed by only 0.007", well within the confidence bounds, so we can conclude that the difference is not statistically significant.

It should be kept in mind at all times that "confidence" intervals such as this one relate only to the precision, or reproducibility, of the measurements. They do not reveal anything about the accuracy of the analysis.

So, how do we establish accuracy? By comparing the results obtained using the procedure being tested to those obtained with established techniques. In this case, the agreement obtained between the means of the Foucault and halo balance measurements for the outer zones of the mirror is a confirmation of the accuracy of the halo balancing measurement.

Although we are primarily interested in using the halo balance readings to add a zone at the extreme edge of the mirror, there is also some potential value in being able to measure very narrow zones within the interior of the mirror using masks of the required diameter. Unfortunately, near the center of the mirror the differences between the means for the halo balancing and Foucault measurements are too large to be considered inconsequential. However, those differences may be due to the gaps between the ring masks and the mirror resulting from the concavity of the mirror. Those gaps increase progressively toward the center of the mirror where the observed deviations are largest. I should also note that the data for the inner zones was obtained first and my own ability to judge the balance point of the diffraction halo may have improved as the data was being obtained. At some future date, I hope to repeat this experiment using masks that stick to the surface of the mirror. For now, I'll leave the idea of making halo balance measurements of very narrow zones on the interior of the mirror using ring masks for later investigation.

While using halo balance measurements to add an extreme edge zone to the Foucault analysis of a mirror has been shown here to be accurate when made by an analytical chemist who is trained to be an objective observer, making the halo balancing measurements visually is not easy. Visually determining the balance points for the left and right sides of the diffraction halo is considerably more difficult than determining the Foucault zonal balance points. The difficulty of judging the halo balance point visually is so great that I question whether most amateurs should use this technique, particularly those who have any tendency to bias their own measurements.

The difficulty inherent in judging the halo balance position is reflected in the standard deviation of the readings which, on average, were about 8 times as large as those associated with the zonal Foucault readings. The statistical hypothesis test known as the F-test can be used to show that a factor of 2.2 difference in standard deviations can be considered significant at the 95% confidence level in this case. There is consequently no doubt that the precision obtained with halo balancing is indeed worse than that obtained from the Foucault test.

To partially compensate for the poorer precision (repeatability) inherent in halo balance measurements, 12 replicate readings were made in determining halo balance positions as compared to six readings obtained for the Foucault zonal nulls throughout this work.

Here are a few tricks that I found helpful for visually judging halo balance positions. 1) Work in a dark room and wait for your eyes to become dark adapted. 2) Use a viewing telescope. 3) Frequently pull the knife edge laterally out of the beam and then move it in again just far enough to extinguish the light from the area of the mirror inside the edge. 4) Force your eyes to move over the edge of the mirror in a circular fashion at a rate of about 1 full rotation every 2 seconds. 5) When you think you've found the balance point, look away into the darkness for a couple of seconds, then look again. You need to be extremely choosy about what you consider to be a good balance and to take your time. Halo balancing is certainly not recommended to anyone who lacks patience or observing skills or who is otherwise uncomfortable with their ability to find nulls in the Foucault test.

The inferior precision (repeatability) of the halo balance readings as compared to Foucault readings does give us one small advantage. Because the ultimate precision of the extended Foucault analysis is essentially determined by the precision of the halo balance readings, we can obtain approximate confidence bounds for the extended Foucault analysis by simply substituting the upper and lower confidence bounds calculated from the halo balance measurements one at a time into one of the Foucault data reduction procedure. The resulting confidence bounds can be regarded as having approximately the same confidence level (~95%) as the bounds calculated from the halo balance measurements. The confidence level of the results cannot be considered to be exactly 95% because we are ignoring the small contribution that would be made by the standard deviations of the Foucault readings at each of the interior zones. However, the fact that we don't know the exact confidence level has little impact on the utility of the results in this application, as it matters little whether we are comparing results at the 96% or 94% confidence level in this application.

While I did not make an exhaustive attempt to optimize the experimental conditions used for halo balance measurements, I did try to improve the precision of the measurements by doubling the slit width and by throwing the viewing telescope out of focus. Neither produced a noticeable improvement in the ease or precision of halo balancing. Optimization of some other experimental parameter might lead to halo balance measurements that are easier to make and more precise. In particular, it seems likely that a photometric procedure using a digital camera or camcorder could prove to be vastly superior in this regard.

G. Results Obtained Using the Halo Balance Extension to the Foucault Test

1. Diagonal 1 of Mirror A

The mean halo balance point for the extreme edge of diagonal 1 of Mirror A was found to be 0.598" farther from the mirror than the null for zone 5, through which it was referenced to the original Foucault data appearing in Appendix 1. The referencing process and its impact on the results is discussed in Appendix 2. The standard deviation of the halo balance readings for Mirror A was 0.066". The P-V error and Strehl ratio for Mirror A using the mean and the 95% confidence interval bounds of the halo balance readings are compared with the usual Foucault results in the table below.

Foucault and Halo Balance Results for Diagonal 1 of Mirror A.

Value Used as the Knife Edge Offset for the Extreme Edge Zone:	P-V Wavefront Error of Mirror A, Diagonal 1	Strehl Ratio of Mirror A, Diagonal 1
None (Standard Foucault):	1/3.0 wave	0.875
Mean of 12 halo balance readings:	1.15 waves	0.693
Upper 95% confidence bound calculated from halo balance readings:	1.23 waves	0.673
Lower 95% confidence bound calculated from balance readings:	1.06 waves	0.711

Hence, for Mirror A, using halo balancing:

~95% P-V error confidence Interval: 1.06 to 1.23 waves;
~95% Strehl ratio confidence interval: 0.673 to 0.711.

The use of the halo balancing extension to the Foucault analysis drops the P-V wave rating for diagonal 1 of Mirror A from 1/3 wave to around 1.1 waves and drops the Strehl ratio from 0.875 to 0.693. The statistical significance of these changes is evident from comparison of the normal Foucault results with the confidence bounds obtained using halo balancing. Halo balancing has therefore lead to a major revision of the Foucault results for Mirror A.

The precision of the halo balance readings reflected in these confidence intervals is obviously good enough to make the use of halo balancing worthwhile even though it is considerably worse than the precision of the Foucault readings.

The difference in the wave rating for Mirror A brought about by adding a zonal measurement at the extreme edge should not be exclusively attributed to zonal errors at the edge. Another factor can contribute to this difference. In particular, introducing a measurement at the extreme edge of the mirror forces the data reduction procedure to stop assuming that the region beyond the center of the outer Couder zone does whatever is needed to minimize the effect of errors on the interior of the mirror.

2. Diagonal 1 of Mirror B

The mean halo balance point for the extreme edge of diagonal 1 of Mirror B was found to be 0.205" farther from the mirror than the null for zone 1, through which it was referenced to the original Foucault data set appearing in Appendix 1. The referencing process itself is discussed in Appendix 2. The standard deviation of the halo balance readings for Mirror B was 0.022". The P-V error and Strehl ratio for Mirror B using the mean and the 95% confidence interval bounds of the halo balance readings are compared with the usual Foucault results in the table below.

Foucault and Halo Balance Results for Diagonal 1 of Mirror B.

Value Used as the Knife Edge Offset for the Extreme Edge Zone:	P-V Wavefront Error of Mirror B, Diagonal 1	Strehl Ratio of Mirror B, Diagonal 1
None (Standard Foucault):	1/11.4 waves	0.971
Mean halo balance reading:	1/6.8 waves	0.961
Upper 95% confidence bound calculated from halo balance readings:	1/5.5 waves	0.951
Lower 95% confidence bound calculated from halo balance readings:	1/8.8 waves	0.967

Hence, for Mirror B, using halo balancing:

~95% P-V error confidence Interval: 1/5.5 to 1/8.8 waves;
~95% Strehl ratio confidence interval: 0.951 to 0.967.

If you compare the confidence bounds obtained using halo balancing to the normal Foucault results, it is evident that halo balancing produces changes in both the P-V error and the Strehl ratio that are statistically significant, though the effect on the Strehl ratio is small in absolute terms. Halo balancing has therefore lead to a significant revision of the Foucault results for Mirror B.

In this respect, it should be noted that Mirror B has an excellent edge according to all of the other tests applied here, yet measuring the extreme edge as a zone using halo balancing has produced a statistically significant impact on the results. Though the precision of halo balancing measurements is arguably worse than for normal Foucault measurements, it is clearly good enough for the technique of halo balancing to be considered useful.

3. Diagonal 1 of Mirror C

It was not possible to balance the left and right sides of the diffraction halo of Mirror C, even going out as far as 2 inches beyond the null position for zone 5. The side of the diffraction halo opposite the knife edge was brightly illuminated as the knife edge was moved away from the mirror starting from the null position for zone 5. So, the attempt at halo balancing at least tells us that the extreme edge of Mirror C is turned down rather sharply.

Unfortunately, the inability to bring the left and right sides of the diffraction halo into balance for Mirror C made it impossible to obtain halo balance measurements for Mirror C. This reveals a limitation of the halo balancing procedure. Specifically, that it can detect but not measure a sharply turned edge.

In many cases, the most practical solution to a narrow, sharply turned edge is to simply mask it off in the telescope using a ring mask whose radius is chosen to just obscure the sharply turned edge. It would be useful as a final bench test of a mirror that is destined to be masked in this way to obtain a halo balance measurement at the working edge of the mirror created by the chosen ring mask. That would sidestep the inability of halo balancing to yield a measurement on sharply turned edges and would lead to a more relevant Foucault analysis of the mirror in its working form.

H. The Theoretical Foundation of Halo-Balancing Revisited

After developing the halo balance test, I was eventually able to obtain a copy of an article by Banerji (Banerji, Sudhansukumar, Astrophys. J., 48,50-58 (1918)) in which the underlying theory of edge diffraction by a concave mirror bounded with straight, parallel edges is explored in detail. The English translation of Texereau's book (p68) cited Banerji's article as proposing a test based on the Rayleigh diffraction ring. However, the Banerji article makes no mention of a test and does not propose any particular application of its findings to mirror testing.

Texereau might have meant that Banerji's article lays the theoretical groundwork for such a test; I suspect that this got shaded to mean "Banerji proposes a test" in the English translation of the book.

Nonetheless, the Banerji article was apparently the first to point out that the intensities of the left and right edges are imbalanced at longitudinal knife edge positions both in front of and behind the "focal plane" in the Foucault test, achieving equality only at the focal plane. Banerji also showed mathematically that the diffraction theory put forth by Lord Rayleigh agrees with this behavior.

While Banerji referred to "focus" and the "focal plane" within the context of the Foucault test, that position is today more commonly referred to as testing at the radius of curvature. And, the axis crossing point in the Foucault test is not truly the "focus" for the light rays coming from a small portion of a concave mirror, as F.L.O. Wadsworth pointed out in Astronomy, 10, 337-348, (1902). However, the slightly inaccurate use of the terms focal plane and focus does not discredit Baneriji's work in the slightest.

VIII. The Couder (Knife Edge Diffraction) Test for Edge Condition

A. Background

Texereau, 2nd Edition, page 69, relates the fundamental information needed to use the knife edge diffraction pattern to gauge the magnitude of mirror defects and credits the test to Andre Couder. A shift in the shadow of the knife edge equal to the distance to the first dark band of the diffraction pattern represents a wavefront error of about 3/4 wave. This gives us a measurable standard with which to directly assess the magnitude of errors at the edge of a mirror in a semi-quantitative way.

The contrast seen in this diffraction pattern diminishes markedly at larger slit sizes, but is adequate at the 30-50 micron width normally employed in Foucault testing, if the image is photographed and later analyzed.

B. Experimental

The wavelength of the source used was approximately 486 nm, about 12% shorter than the usual 550 nm. No correction for this difference was made since such a correction lies well within the expected error limits of this test.

The central ROC of each mirror was found and the knife edge was offset 0.500" toward the mirror. The knife was then moved laterally to shadow slightly more than 50% of the mirror's area.

No attempt was made to perform the test visually. Instead, the diffraction patterns seen for all 3 test mirrors were photographed using an Olympus C3040 digital camera for subsequent analysis. The resulting images had their contrast enhanced digitally and were displayed on a computer monitor for measurement.

C. Results

1. Mirror A

Analysis of the knife edge diffraction pattern showed that Mirror A has a turned down edge about 1/4” wide introducing a wavefront error of approximately 1/2 wave P-V. Unfortunately, the roughness of this mirror introduces considerable uncertainty into the measurement and the error introduced by the turned edge could be significantly larger than this.

2. Mirror B

This test showed no evidence of a turned edge on Mirror B. An error of 1/8 wave is believed to be the minimum detectable in this case.

3. Mirror C

The edge is turned down by approximately 1/4 wave. The exact position at which the turn begins is difficult to judge, but measurements made from the photograph place it at around 1/10” to 1/20” from the edge.

IX. The Eyepiece Test for Astigmatism

A. Experimental

The eyepiece test for astigmatism was conducted in a manner similar to that described by Texereau, 2nd Edition (p80).

Positioning and aiming all of the elements used in the eyepiece test can be extremely difficult. Adding to that difficulty is the fact that the image of the artificial star described by Texereau is extremely dim and becomes invisible only a short distance from focus. To address these problems, a much brighter source was used. Also, a laser collimator was employed to reverse-align the various optics involved in the test. The details of these improvements are discussed below.

The improved source was constructed using a laser pointer and a 1/2” diameter 1” focal length lens. The lens was mounted in a 1” length of 1/2” I.D. rubber tubing and affixed to the end of the 1/2” diameter laser pointer. The resulting source produces a bright, slightly diverging but well defined beam that spreads to a diameter of about 4” at a working distance of 60”. This source was placed off axis and aimed so as to illuminate a 5/16” diameter polished steel ball bearing. The working distance of 60” was calculated to make the 1/2” lens of the source appear to be at most 1/2 degree in angular size as viewed from the position of the ball bearing. In this situation, the diameter of the artificial star produced by reflection of the source light in the ball bearing is at most 1/300th of the diameter of the ball bearing, or 2.68 microns. By comparison, the diameter of the Airy disk at the focal plane is 1.22 x 2 x .56 microns x f-ratio. For an f-ratio of 4, the Airy disk would be 5.46 microns in diameter. The artificial star produced by this setup is thus at most 1/2 the size of the Airy disk for the lowest f-ratio mirror we are likely to test.

The eyepiece was mounted in a clamping holder made by drilling a 1.25” hole in a piece of 1X pine (0.75” thick). The eyepiece was masked against glare from the source by a divider placed between the eyepiece and the ball bearing. In addition, a tube made of black flocking paper and sized to slip over the end of the eyepiece was used to further reduce glare.

The ball bearing used was mounted 1.75” from the optical axis of the eyepiece. This lead to a very slight hint of astigmatism in the test, but it is easily recognized as being due to the test setup when the optic under test is rotated. The high sensitivity of the test helps in this matter, as any significant amount of astigmatism is strikingly evident compared to the weak indication of astigmatism introduced by this arrangement.

A common laser collimator can be used to greatly simplify the alignment of the mirror, ball bearing, eyepiece, and light source. Here’s how. First, substitute the laser collimator for the eyepiece. Turn off the light source for the test and turn on the laser collimator. Find the return beam coming from the mirror under test and aim it at the ball bearing using the mirror stand adjustments. With lights out, find the diverging beam of light reflected from the ball bearing and aim it back in the direction of the light source to be used for the test by shifting the position of the test stand horizontally and by raising or lowering the light source. Turn off the laser collimator and replace it with the eyepiece. Turn the test light source back on. If the focal plane of the eyepiece is at or near the focal point of the star image, you should be able to see the image of the artificial star. If not, try a longer focal length eyepiece to find and center the image of the star, then switch back to the eyepiece needed for the test.

B. Results for Mirrors A through C

As noted earlier, the rather large differences between the Foucault results for the two diameters of Mirror B hints at the possibility of astigmatism in this mirror. However, no detectable astigmatism was revealed by this test for ANY of the three test mirrors. Texereau indicates that even 1/10th wave of astigmatism is easily detected using this test.

X. Summary of Results

A. Summary of Results for Mirror A

Mirror A showed no large scale strain under crossed polarizers and would not be expected to show significant changes of figure as a function of temperature following equilibration.

Mirror A scattered more light in the laser test than either Mirrors B or C and much more than the float glass reference, which strongly suggests that this mirror was not completely polished out. There is also a chance that this mirror was made from an inferior grade of glass that has a higher level of defects at best polish than the other two mirrors.

Ronchi testing showed Mirror A to be very rough, with a turned down edge about 5/16" wide. I would give this mirror a failing grade based on the roughness alone.

Zonal Foucault testing along diameter 1 yielded a P-V error of 1/3 wave on the wavefront and a Strehl ratio of 0.875, while testing on diameter 2 yielded a P-V error of 1/4 wave and a Strehl ratio of 0.924. The average Strehl ratio for the two diameters was 0.90. If the usual zonal Foucault test were the only test conducted on this mirror, it would get an undeserved passing grade.

Using the Halo Balance extension to the Foucault test, the P-V error rating for diameter 1 of Mirror A is downgraded to 1.15 waves and the corresponding Strehl ratio is likewise downgraded to 0.693. The changes in both parameters were shown to be statistically significant, in spite of the poorer precision of halo balance readings as compared to the usual Foucault readings.

The Couder test showed that Mirror A has a turned down edge approximately 1/4" wide of approximately 1/2 wave P-V on the wavefront, though the roughness of this mirror places the exact magnitude of the turned edge in doubt.

The eyepiece test for astigmatism showed no evidence of astigmatism on this mirror.

Recommendation: This mirror should go back to the polishing stage to be polished out, to have the surface roughness reduced, and to be refigured.

B. Summary of Results for Mirror B

Mirror B showed a slight Maltese cross pattern under crossed polarizers, though I am doubtful that it will show significant changes of figure as a function of temperature following equilibration. However, testing of this optic over a wide range of temperatures might provide much needed information for future mirror makers on whether the level of strain seen with this mirror is in fact acceptable in practice.

The laser test showed that Mirror B was either not polished out or was made from a grade of glass that has a higher level of defects when polished than float glass.

Ronchi testing showed Mirror B to be smooth. To the extent it can be judged with the Ronchi test, this mirror has a perfect edge. However, the Ronchi test showed a somewhat abrupt change in the curvature of the mirror at about 1.125” in from the edge, indicating a slightly raised zone.

Zonal Foucault testing of Mirror B along diameter 1 yielded a P-V error of 1/11th wave on the wavefront and a Strehl ratio of 0.971, while testing on diameter 2 yielded a P-V error of 1/6 wave and a Strehl ratio of 0.925. The average Strehl ratio for the two diameters was 0.95.

The Halo Balance extension of the Foucault test changed the P-V error rating for diameter 1 of Mirror B from 1/11th wave to 1/6.8 waves, but changed the Strehl ratio only slightly, dropping it from 0.971 to 0.961. The changes in both parameters were shown to be statistically significant, in spite of the poorer precision of halo balance readings as compared to the usual Foucault readings.

The Couder test showed no evidence of a turned edge on Mirror B. It is believed that the limit of detection in this instance is about 1/8 wave.

The eyepiece test for astigmatism showed no evidence of astigmatism on this mirror.

Recommendation: Mirror B is good enough to go into a telescope as it is. Only a modest improvement for low contrast detail in planetary images could be achieved by refiguring this mirror to a higher degree of perfection.   For most astronomical objects this mirror will be indistinguishable from perfect.

C. Summary of Results for Mirror C

Mirror C showed a pronounced Maltese cross pattern under crossed polarizers. It is uncertain whether Mirror C will show significant changes of figure as a function of temperature following equilibration. Testing of this optic over a wide range of temperatures could provide much needed information for future mirror makers on whether the level of strain seen with this mirror is acceptable in practice.

The laser test indicated that Mirror C was either not polished out or was made from a grade of glass that has a higher level of defects when polished than float glass. Visually, this mirror has about the same level of polish as mirror B.

Ronchi testing showed that this mirror was generally smooth, but it also revealed a narrow turned-down edge approximately 0.1” wide. The Ronchi test also showed a barely perceivable raised area in the center just under 1” in diameter.

Zonal Foucault testing of Mirror C along diameter 1 yielded a P-V error of 1/22 waves on the wavefront and a Strehl ratio of 0.994, while testing on diameter 2 yielded a P-V error of 1/10 waves and a Strehl ratio of 0.989. The average Strehl ratio for the two diameters was 0.992.

The Halo Balance extension of the Foucault test revealed that this mirror has a sharp turned-down edge, but the two sides of the diffraction halo could not be balanced for this mirror and consequently a quantitative measurement could not be obtained.

The Couder test indicated that Mirror C has a turned edge between 0.1” and 0.2” wide with a magnitude of about 1/4 wave on the wavefront.

The eyepiece test for astigmatism showed no evidence of astigmatism on this mirror.

Recommendation: This mirror is good enough to go into a telescope with the outer 0.2” masked off. Using such a mask, the performance of this mirror will be indistinguishable from perfect for all astronomical objects, as its figure is superb except at the extreme edge.

Appendix 1. Zonal Foucault Readings and Related Data.

Mirror A.

Tester: Wm. D. Hanagan, Jr., Ph.D.
Optical Diameter = 7.953”
ROC at center = 85.0”
Fixed Source
Unit of measure = inch

Mirror A Diameter 1

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Inner radius	0.61	1.74	2.34	2.82	3.25	3.64
Outer radius	1.74	2.34	2.82	3.25	3.64	3.98*
Area weighted zone center (r)	1.30	2.04	2.58	3.04	3.45	3.81
Reading 1	-0.5177	-0.5271	-0.5085	-0.4550	-0.4420	-0.3925
Reading 2	-0.5147	-0.5278	-0.5087	-0.4599	-0.4441	-0.4029
Reading 3	-0.5167	-0.5279	-0.5069	-0.4540	-0.4450	-0.4029
Reading 4	-0.5163	-0.5318	-0.5087	-0.4644	-0.4439	-0.4075
Reading 5	-0.5152	-0.5278	-0.5092	-0.4607	-0.4415	-0.4006
Reading 6	-0.5132	-0.5321	-0.5063	-0.4597	-0.4429	-0.4001

Mirror A Diameter 2

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Inner radius	0.61	1.74	2.34	2.82	3.25	3.64
Outer radius	1.74	2.34	2.82	3.25	3.64	3.98*
Area weighted zone center (r)	1.30	2.04	2.58	3.04	3.45	3.81
Reading 1	-0.5269	-0.5263	-0.4982	-0.4672	-0.4397	-0.3930
Reading 2	-0.5189	-0.5234	-0.4993	-0.4580	-0.4402	-0.3942
Reading 3	-0.5219	-0.5273	-0.4957	-0.4610	-0.4360	-0.3893
Reading 4	-0.5219	-0.5260	-0.4963	-0.4628	-0.4397	-0.3958
Reading 5	-0.5211	-0.5263	-0.4942	-0.4583	-0.4390	-0.3930
Reading 6	-0.5223	-0.5242	-0.4971	-0.4632	-0.4387	-0.3947

For the rating of this mirror, see the section of the report titled Summary of Results.

* Because the optical radius of the mirror was smaller, it was used as the outer radius of the last zone in place of the measurement taken from the mask.

Mirror B.

Tester: Wm. D. Hanagan, Jr., Ph.D.
Optical Diameter = 7.972”
ROC at center = 80.4”
Fixed Source
Unit of measure = inch

Mirror B Diameter 1

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Inner radius	0.61	1.74	2.34	2.82	3.25	3.64
Outer radius	1.74	2.34	2.82	3.25	3.64	3.99*
Area weighted zone center (r)	1.30	2.04	2.58	3.04	3.45	3.82
Reading 1	-0.8535	-0.8181	-0.7982	-0.7638	-0.7141	-0.6949
Reading 2	-0.8533	-0.8169	-0.7965	-0.7640	-0.7120	-0.6958
Reading 3	-0.8552	-0.8180	-0.7945	-0.7648	-0.7142	-0.6936
Reading 4	-0.8543	-0.8208	-0.7995	-0.7641	-0.7158	-0.6928
Reading 5	-0.8542	-0.8240	-0.7987	-0.7658	-0.7209	-0.6954
Reading 6	-0.8576	-0.8222	-0.7978	-0.7627	-0.7160	-0.6942

Mirror B Diameter 2

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6
Inner radius	0.61	1.74	2.34	2.82	3.25	3.64
Outer radius	1.74	2.34	2.82	3.25	3.64	3.99*
Area weighted zone center (r)	1.30	2.04	2.58	3.04	3.45	3.82
Reading 1	-0.8912	-0.8480	-0.8327	-0.7938	-0.7432	-0.7171
Reading 2	-0.8903	-0.8501	-0.8303	-0.7942	-0.7477	-0.7208
Reading 3	-0.8918	-0.8529	-0.8279	-0.7940	-0.7457	-0.7200
Reading 4	-0.8871	-0.8552	-0.8279	-0.7967	-0.7432	-0.7179
Reading 5	-0.8869	-0.8522	-0.8283	-0.7961	-0.7418	-0.7181
Reading 6	-0.8883	-0.8562	-0.8282	-0.7967	-0.7458	-0.7190

For the rating of this mirror, see the section of the report titled Summary of Results.

* Because the optical radius of the mirror was smaller, it was used as the outer radius of the last zone in place of the measurement taken from the mask.

Mirror C.

Tester: Wm. D. Hanagan, Jr., Ph.D.
Optical Diameter = 5.945”
ROC at center = 64.6”
Fixed Source
Unit of measure = inch

Mirror C Diameter 1

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
Inner radius	.050	1.40	1.91	2.32	2.67
Outer radius	1.40	1.91	2.32	2.67	2.97*
Area weighted zone center (r)	1.05	1.67	2.12	2.50	2.82
Reading 1	-0.7462	-0.7095	-0.6897	-0.6641	-0.6377
Reading 2	-0.7446	-0.7096	-0.6908	-0.6639	-0.6386
Reading 3	-0.7502	-0.7120	-0.6902	-0.6613	-0.6358
Reading 4	-0.7509	-0.7090	-0.6934	-0.6637	-0.6352
Reading 5	-0.7533	-0.7124	-0.6913	-0.6650	-0.6377
Reading 6	-0.7560	-0.7109	-0.6898	-0.6622	-0.6415

Mirror C Diameter 2

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5
Inner radius	.050	1.40	1.91	2.32	2.67
Outer radius	1.40	1.91	2.32	2.67	2.97*
Area weighted zone center (r)	1.05	1.67	2.12	2.50	2.82
Reading 1	-0.7659	-0.7240	-0.7019	-0.6719	-0.6476
Reading 2	-0.7641	-0.7221	-0.7003	-0.6760	-0.6468
Reading 3	-0.7661	-0.7228	-0.7043	-0.6728	-0.6478
Reading 4	-0.7718	-0.7222	-0.7010	-0.6710	-0.6463
Reading 5	-0.7697	-0.7228	-0.7005	-0.6723	-0.6484
Reading 6	-0.7643	-0.7233	-0.7022	-0.6736	-0.6475

For the rating of this mirror, see the section of the report titled Summary of Results.

* Because the optical radius of the mirror was smaller, it was used as the outer radius of the last zone in place of the measurement taken from the mask.

Appendix 2. The Procedure Used for Referencing Halo Balance Readings with Zonal Foucault Readings Taken Earlier

The test rig is often unavoidably moved between obtaining the original Foucault readings for a mirror and the readings of the halo balance positions. Mathematically, moving the rig between tests shifts the new set of readings by a constant equal to the distance the rig is moved along the test axis, known as the offset. To allow the two sets of readings to be compared and used together in the same data reduction procedure, the second data set must be referenced to the first by subtracting the value of the offset. Here, the tester offset was measured by repeating the Foucault readings for one zone and finding the difference between the means obtained at the two different tester positions.

When careful statistical comparisons of results are to be made, it is usually preferable to avoid referencing the data, since the random error in measuring the offset can combine with the random error in the measurements to be referenced.

However, the standard deviations for the halo balance readings were, on average, about 8 times as large as those associated with the zonal Foucault readings. The squares of the standard deviations are additive here, so the typical increase in the standard deviation of a set of halo balance readings that would be produced by referencing would only be about ((8*2)+1*2)*1/2)/8, or about 0.78%.

The statistical hypothesis test known as the F-test can be used to show that two standard deviations would need to differ by a factor of 2.2 or more to be considered significantly different at a 95% confidence level for the number of measurements used here. The 0.78% increase in the standard deviation of the halo balance measurements brought about by referencing is hence far too miniscule to be considered significant. Consequently, the effects of referencing the data in this case can be considered insignificant.